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Seesaw Mechanism in Warped Geometry

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Abstract

We show how the seesaw mechanism for neutrino masses can be realized within a five dimensional (5D) warped geometry framework. Intermediate scale standard model (SM) singlet neutrino masses, needed to explain the atmospheric and solar neutrino oscillations, are shown to be proportional to $M_{\text{Pl}} \exp((2c - 1)\pi kR)$, where c denotes the coefficient of the 5D Dirac mass term for the singlet neutrino which also has a Planck scale Majorana mass localized on the Planck-brane, and $kR \approx 11$ in order to resolve the gauge hierarchy problem. The case with a bulk 5D Majorana mass term for the singlet neutrino is briefly discussed.

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1 Introduction

A particularly intriguing resolution of the gauge hierarchy problem is provided by a setting based on five dimensional (5D) warped geometry [1] (see also [2]). Without invoking supersymmetry it is possible to derive the 'low energy' TeV scale from 5D Planck scale quantities. Indeed, it may even be possible to derive even smaller scales, such as $\text{TeV}^2/M_{\text{Pl}} \sim 10^{-3} \text{ eV}$ [3] which may shed some light in the understanding of the observed vacuum energy density. The warped framework has some other interesting features. It sheds new light on fermion mass hierarchies and mixings [4–6], and also allows one to accommodate the observed solar and atmospheric neutrino oscillations through dimension five operators, without invoking any additional fields beyond those present in the SM [7]. By introducing SM singlet fermions the observed neutrinos can turn into light Dirac particles [8, 9]. The approach seems to be consistent with the attractive idea of grand unification [10]. Last but by no means least, this approach can be experimentally tested, hopefully at the LHC. In particular, the first KK excitations of the SM particles are expected to lie in the multi (7-10) TeV range [11–14]. In the presence of brane-localized kinetic terms the KK scale may be somewhat lower [15]. A left-right symmetric gauge group in the bulk may also bring down the KK scale to a few TeV [16].

In this paper we investigate how the four dimensional seesaw mechanism can be incorporated within the warped setting. This means that one should understand how an intermediate mass scale for the SM singlet neutrinos arises, starting with Planck scale quantities. We show how this works out by introducing in particular 5D Dirac masses for the SM singlet fields, in addition to the Majorana masses. It remains to be seen if the appearance of an intermediate mass scale for 'right handed' neutrinos can be exploited to yield not only the required light neutrino masses but to also realize the observed baryon asymmetry via leptogenesis [17].

2 KK reduction with a Majorana mass term

We take the fifth dimension to be an S_1/Z_2 orbifold with a negative bulk cosmological constant, bordered by two 3-branes with opposite tensions and separated by distance R . Einstein's equations yield the non-factorizable metric [1]

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad \sigma(y) = k|y| \quad (1)$$

which describes a slice of AdS_5 . The 4-dimensional metric is $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$, k is the AdS curvature related to the bulk cosmological constant and brane tensions, and y denotes the fifth coordinate. The AdS curvature and the 5D Planck mass M_5 are both assumed to be of order $M_{\text{Pl}} = 1.2 \times 10^{19} \text{ GeV}$. The AdS warp factor $e^{-\pi k y}$ generates an exponential hierarchy of energy scales. If the brane separation is $kR \simeq 11$, the natural scale at the negative tension brane, located at $y = \pi R$, is of TeV-size, while the scale at the brane at $y = 0$ is of order M_{Pl} .

We consider the fermionic action on the warped background (1)

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \sqrt{-G} \left(\bar{\Psi} i E_a^M \gamma^a (\partial_M + \omega_M) \Psi - m_D \bar{\Psi} \Psi - m_M \bar{\Psi} \Psi^c \right), \quad (2)$$

where E_a^M is the fünfbein and $\gamma^a = (\gamma^\mu, \gamma^5)$ represent the Dirac matrices in flat space. The index M refers to objects in curved 5D space, the index a to those in tangent space. The spin connection related to the metric (1) is found to be $\omega_M = (\frac{1}{2} \sigma' e^{-\sigma} \gamma^5 \gamma_\mu, 0)$, with $\sigma' = d\sigma/dy$. $\Psi^c = C_5 \gamma^0 \Psi^*$ is the charge conjugated spinor.

Fermions in 5D are non-chiral. Chirality in the 4D low energy effective theory is restored by the orbifold boundary conditions. Fermions have two possible transformation properties under the Z_2 orbifold symmetry, $\Psi(-y)_\pm = \pm i \gamma^5 \Psi(y)_\pm$, depending on whether the left- or right-handed components are chosen to be even. Thus, $\bar{\Psi}_\pm \Psi_\pm$ is odd under Z_2 , and the Dirac mass parameter, which is also odd, can be parametrized as $m_D = -c\sigma'$.³ The bilinear $\bar{\Psi}_\pm \Psi_\pm^c$ is even, resulting in an even Majorana mass m_M . The Majorana mass can have bulk and boundary contributions. The boundary mass terms are restricted only by 4D Lorentz invariance and one could think of choosing them differently for the left- and right-handed components of the Dirac spinor. However, boundary mass terms are only felt by the even components. The odd components do have only derivative couplings to the boundary.

In the following we perform the KK reduction of the action (2) to four dimensions. Without the Majorana mass m_M this has first been discussed in ref. [8] (see also [4]). Using the warped metric (1) and defining $\hat{\Psi} = e^{-2\sigma} \Psi$ we obtain

$$S = \int d^4x \int_{-\pi R}^{\pi R} dy \left(\bar{\hat{\Psi}} (i e^\sigma \gamma^\mu \partial_\mu + i \gamma^5 \partial_5) \hat{\Psi} - m_D \bar{\hat{\Psi}} \hat{\Psi} - m_M \bar{\hat{\Psi}} \hat{\Psi}^c \right). \quad (3)$$

We decompose the 5D fields as

$$\hat{\Psi}_{L,R}(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Psi_{L,R}^{(n)}(x^\mu) f_{L,R,n}(y), \quad (4)$$

where $\Psi_{L,R} = \pm i \gamma^5 \Psi_{L,R}$. For non-vanishing m_M the spectrum of KK states is no longer vector-like. Instead, it consists of an infinite tower of Majorana fermions with masses m_n . Requiring that after y integration the action (3) reduces to the usual action of massive Majorana fermions in four dimensions, the wave functions $f_{L,R,n}$ must obey the conditions

$$\begin{aligned} -m_M f_{L,n} - (\partial_5 + m_D) f_{R,n}^* &= -m_n e^\sigma f_{L,n}^* \\ -m_M f_{R,n}^* + (\partial_5 - m_D) f_{L,n} &= -m_n e^\sigma f_{R,n}. \end{aligned} \quad (5)$$

³The minus sign in the definition ensures that the meaning of c matches with refs. [4, 5], which use a different signature of the metric.

To arrive at these expressions we have used the 4D Majorana condition $\bar{\Psi}_R^{(n)} = \Psi_L^{(n)}$. For $m_M = 0$ we reproduce the results of ref. [8]. The normalization conditions read

$$\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy e^\sigma (f_{L,m}^* f_{L,n} + f_{R,m}^* f_{R,n}) = \delta_{mn}. \quad (6)$$

Notice that for non-vanishing Majorana mass $f_{L,n}$ and $f_{R,n}$ are no longer complete sets of functions by there own. If m_M is real, the eqs. (5) can be split into real parts

$$\begin{aligned} -m_M \text{Re} f_{L,n} - (\partial_5 + m_D) \text{Re} f_{R,n} &= -m_n e^\sigma \text{Re} f_{L,n} \\ -m_M \text{Re} f_{R,n} + (\partial_5 - m_D) \text{Re} f_{L,n} &= -m_n e^\sigma \text{Re} f_{R,n} \end{aligned} \quad (7)$$

and imaginary parts

$$\begin{aligned} -m_M \text{Im} f_{L,n} + (\partial_5 + m_D) \text{Im} f_{R,n} &= m_n e^\sigma \text{Im} f_{L,n} \\ m_M \text{Im} f_{R,n} + (\partial_5 - m_D) \text{Im} f_{L,n} &= -m_n e^\sigma \text{Im} f_{R,n}. \end{aligned} \quad (8)$$

The eqs. (7) and (8) are related by $m_M \rightarrow -m_M$. For a complex Majorana mass eqs. (7) and (8) no longer separate.

For $m_M = 0$ eqs. (7) and (8) allow for a chiral zero mode solution, and the chirality depends on the chosen orbifold boundary conditions. If the Majorana mass term is turned on, the zero mode picks up a mass and becomes a mixture of left- and right-handed states. We still can decouple left- and right-handed states in eqs. (7) and (8) and end up, for instance, with

$$-m_M \text{Re} f_{R,n} - (\partial_5 - m_D) \frac{1}{m_n e^\sigma - m_M} (\partial_5 + m_D) \text{Re} f_{R,n} = -m_n e^\sigma \text{Re} f_{R,n}. \quad (9)$$

This equation is complicated but can be solved numerically. Taking into account the boundary conditions, e.g. $\text{Re} f_{R,n}(0) = \text{Re} f_{R,n}(\pi R) = 0$ for odd right-handed modes, the spectrum of KK masses can be determined. Potential problems arise if $1/(m_n e^\sigma - m_M)$ becomes singular.

A particularly simple case arises if the Majorana mass is confined to a boundary. Then we can build the wave functions from the $m_M = 0$ solutions [4, 8]

$$\begin{aligned} \text{Re} f_{L,n}(y) &= \frac{e^{\sigma/2}}{N_n} \left[J_{-c-1/2} \left(\frac{m_n}{k} e^\sigma \right) + b(m_n) Y_{-c-1/2} \left(\frac{m_n}{k} e^\sigma \right) \right] \\ \text{Re} f_{R,n}(y) &= \frac{e^{\sigma/2}}{N_n} \left[J_{-c+1/2} \left(\frac{m_n}{k} e^\sigma \right) + b(m_n) Y_{-c+1/2} \left(\frac{m_n}{k} e^\sigma \right) \right], \end{aligned} \quad (10)$$

with $b(m_n) = -J_{-c+1/2}(\frac{m_n}{k}\Omega)/Y_{-c+1/2}(\frac{m_n}{k}\Omega)$. The warp factor is defined as $\Omega = e^{\pi k R}$. The Majorana mass shows up only in the boundary conditions. If the Majorana mass is confined to the Planck-brane, i.e. $m_M = d \cdot \delta(y)$, we find

$$\begin{aligned} \text{Re} f_{R,n}(0) - \frac{d}{2} \text{Re} f_{L,n}(0) &= 0 \\ \text{Re} f_{R,n}(\pi R) &= 0, \end{aligned} \quad (11)$$

where we have chosen f_R to be odd. These equations demonstrate the coupling between left- and right-handed states which is introduced by the Majorana mass term. Taking $d = 0$ we recover the result of [8]. The spectrum of KK masses $x_n = m_n/k$ is finally obtained from

$$\begin{vmatrix} J_{-c+1/2}(x_n\Omega) & Y_{-c+1/2}(x_n\Omega) \\ J_{-c+1/2}(x_n) - \frac{d}{2}J_{-c-1/2}(x_n) & Y_{-c+1/2}(x_n) - \frac{d}{2}Y_{-c-1/2}(x_n) \end{vmatrix} = 0. \quad (12)$$

The analogous expressions for the imaginary parts of $f_{L,R}$ are obtained by switching the sign of the Majorana mass.

3 The KK spectrum

For a vanishing Majorana mass the KK spectrum of a bulk fermion consists of a chiral zero mode, which we choose to be left-handed, and a tower of excited vector-like states. The location of the zero mode depends on the bulk Dirac mass [8]

$$f_{L,0}(y) = \frac{e^{-ck|y|}}{N_0}. \quad (13)$$

For $c > 1/2$ ($c < 1/2$) the zero mode is localized near the boundary at $y = 0$ ($y = \pi R$), i.e. at the Planck- (TeV-) brane. The excited KK states are always localized at the TeV-brane.

If we turn on a small Majorana mass, the zero mode picks up a mass. Its wave function receives a non-vanishing odd (right-handed) component. Defining

$$r_{L,R,n} = \frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy e^{\sigma} f_{L,R,n}^* f_{L,R,n}, \quad (14)$$

the even content of the wave function $f_n = (f_{L,n}, f_{R,n})$ is given by

$$r_n = \frac{r_{L,n}}{r_{L,n} + r_{R,n}}. \quad (15)$$

The vector-like pairs of excited states split up. Once the Majorana mass becomes larger than a critical value, the zero mode reaches the KK scale and disappears from the low energy spectrum.

Let us discuss the case of a Majorana mass confined to the Planck-brane in more detail. As long as $d \lesssim \Omega^{-2c}$ there exists an (almost) chiral mode with mass

$$\begin{aligned} x_0\Omega &\approx \frac{d}{2}(1-2c)\Omega^{2c}, & c &\lesssim \frac{1}{2} \\ x_0\Omega &\approx \frac{d}{2}(2c-1)\Omega, & c &\gtrsim \frac{1}{2} \\ x_0\Omega &\approx 0.015 \cdot d, & c &= \frac{1}{2}. \end{aligned} \quad (16)$$

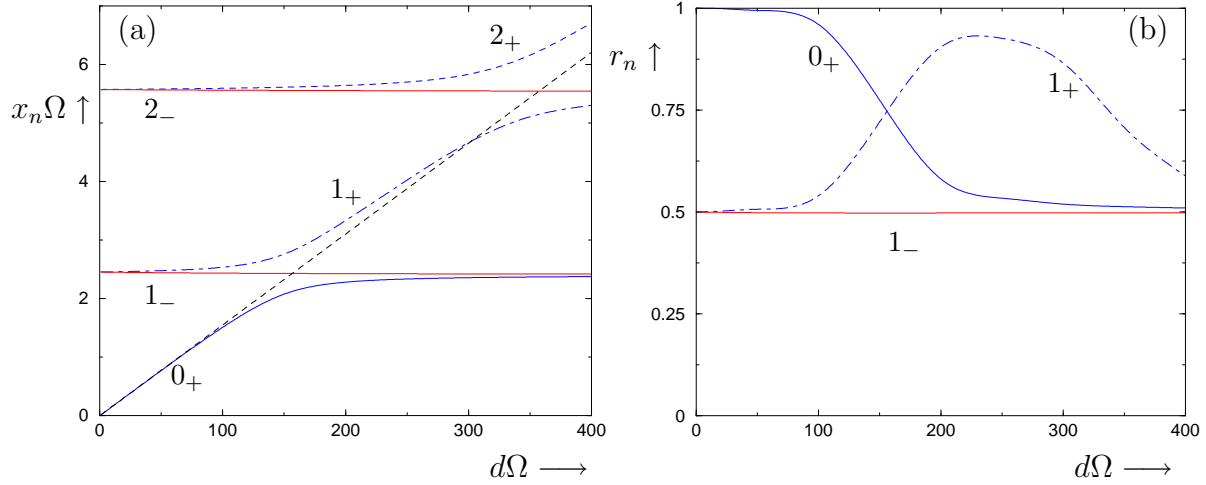


Figure 1: The KK masses (a) and the even content (b) of the lowest KK states as a function of the Majorana mass $m_M = d \cdot \delta(y)$. We have taken $c = 1/2$ and $\Omega = 10^{14}$.

The splitting of the masses of the excited states is proportional to d/x_n . Their overall mass is almost unchanged. This behavior becomes clear from fig. 1a, where we present the lowest KK masses as a function of the Majorana mass d . In this example we have taken the parameters $c = 1/2$ and $\Omega = 10^{14}$. We have labeled the states i_{\pm} depending on whether they arise from eqs. (7) or (8). For $d > 0$ the “zero mode” belongs to eq. (8). At $d\Omega \approx 150$ the mass of 0_+ becomes comparable to the first KK mass, where it saturates, while the mass of 1_+ starts to increase. If the Majorana mass is further increased this phenomenon happens at higher KK levels, i.e. the states n_+ join the KK level $(n+1)$. During this process the masses of the states n_- remain practically constant. For large Majorana masses, in our example $d\Omega \gg 150$ the mass splitting in the KK level formed by n_+ and $(n+1)_-$ is proportional to x_n/d .

It is instructive to study the content of even states among the wave functions, which we present in fig. 1b. For $d = 0$ we have $r(0_+) = 1$, which means that there is truly a chiral zero mode. The excited states are perfect even-odd mixtures, i.e. $r = 1/2$. If a Majorana mass is turned on, $r(n_+)$ changes, while the content of the n_- states is not significantly changed. In the range of d where $x(1_+)$ is rapidly growing, 1_+ becomes an almost pure even state. This means that as the Majorana mass is increasing an almost even state (“chiral state”, of course it is a massive state!) is moving through the KK spectrum.

In fig. 2 we present the wave functions of the 0_+ and 1_+ states. The odd component of 0_+ becomes more and more important as we increase d from 50 to 200. At the same time the even part of 0_+ gets suppressed in the bulk and gets localized towards the TeV-brane like an excited state. The state 1_+ is localized towards the TeV-brane for $d = 100, 400$. For $d = 200$ its even component becomes somewhat delocalized. At the same time the amplitude of the odd component shrinks, as expected from fig. 1b.

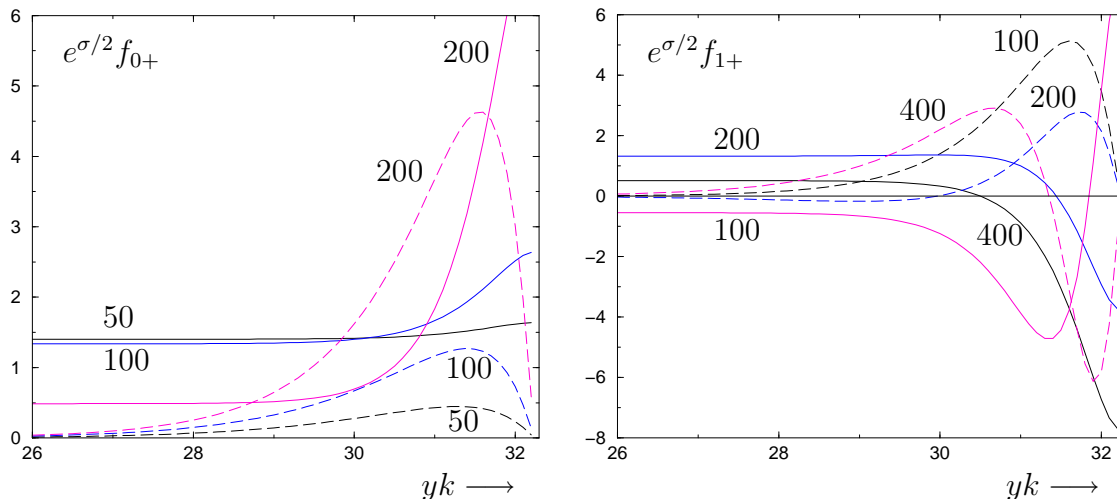


Figure 2: The wave functions of the 0_+ and 1_+ states in the vicinity of the TeV-brane for different values of the Majorana mass ($d = 50, 100, 200$ and $d = 100, 200, 400$). The left- (right-) handed components are shown in solid (dashed) lines. We have taken $c = 1/2$ and $\Omega = 10^{14}$.

Using eqs. (7) and (8), the Majorana mass is included in the KK reduction from the very beginning. This procedure is analogous to our treatment of boundary masses of gauge bosons in refs. [11, 12]. Alternatively, the KK reduction can be done with a vanishing Majorana mass. When the 5D action is integrated over the extra dimension, the Majorana mass term induces additional operators which mix the different KK states. The general mass matrix \mathcal{M} is given by

$$\mathcal{L}_M = (\Psi_L^{(0)}, \Psi_L^{(1)}, \bar{\Psi}_R^{(1)}, \dots) \begin{pmatrix} A_{00} & A_{01} & 0 & \cdots \\ A_{01} & A_{11} & D_1 & \cdots \\ 0 & D_1 & B_{11} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \Psi_L^{(0)} \\ \Psi_L^{(1)} \\ \bar{\Psi}_R^{(1)} \\ \vdots \end{pmatrix}, \quad (17)$$

where D_n is the n th KK mass and

$$\begin{aligned} A_{mn} &= \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} m_M(y) f_{L,m}(y) f_{L,n}(y) \\ B_{mn} &= \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} m_M(y) f_{R,m}(y) f_{R,n}(y). \end{aligned} \quad (18)$$

Note that $f_{L,R,n}$ and $\Psi_{L,R}^{(n)}$ here denote the fields and wave functions obtained with a vanishing Majorana mass, while earlier these symbols were used for the mass eigenstates including the Majorana mass. The zeros in \mathcal{M} follow from the orbifold Z_2 symmetry. For a boundary Majorana mass B_{mn} vanishes.

The advantage of eqs. (7) and (8) is that they diagonalize the infinite dimensional mass matrix \mathcal{M} in a single step. However, it turns out that in many cases simple

finite truncations of \mathcal{M} provide valuable information on the KK spectrum. Let us focus again on the case of a Planck-brane Majorana mass, where B_{mn} vanishes. For $c \gtrsim 0.3$ the mass spectrum can be reliably computed up to the n th KK level by taking into account the states $\Psi^{(0)}$ to $\Psi^{(n)}$. The results rapidly converge if more KK states are included. For small values of the Majorana mass, i.e. as long as $x_0 \ll x_1$ or $A_{00} \ll D_1$, one finds for the former zero mode a mass of

$$x_0 \approx \frac{A_{00}}{k}. \quad (19)$$

The mass splitting of the n th KK level is found to be

$$\Delta x_n \approx \frac{A_{nn}}{k}. \quad (20)$$

For large Majorana masses, $A_{00} \gg D_1$, the mass of the almost even (“chiral”) state, which is moving up the spectrum, is approximately given by A_{00} . In the next section the mass of this state will be identified with the seesaw mass scale. The mass splitting of the first KK level reads

$$\Delta x_1 \approx \frac{A_{11} D_1^2}{(A_{00} + A_{11})^2 k}. \quad (21)$$

For $c \lesssim 0.3$ the former zero mode becomes closely localized towards the TeV-brane. Then the A_{ij} are no longer dominated by A_{00} and eq. (21) receives non-negligible corrections from higher KK modes. For a Majorana mass term on the TeV-brane eq. (21) receives corrections as well.

The truncated mass matrix (17) can be used to study Majorana mass profiles for which eqs. (7) and (8) are not analytically solvable. Let us discuss the case of a homogeneous bulk Majorana mass $m_M(y) = d \cdot k$. For $d \ll \Omega^{-1}, \Omega^{-2c}, 1$, where $c \gtrsim 1/2$, $0 \lesssim c \lesssim 1/2$, $c \lesssim 0$, there is still a light mode, whose mass is given by eq. (19). The mass splittings of the KK levels receive corrections from the non-vanishing B_{mn} . For the first KK level one finds $\Delta x_1 \approx (A_{11} + B_{11})/k$. As long as $c \gtrsim 0$, B_{mn} turns out to be only a tiny correction of order $B_{11}/A_{11} \sim \Omega^{-2c}$. For $d \sim 1$ the mass splitting becomes comparable to the splitting between different KK levels. The pairing of KK states is completely gone. Thus bulk and boundary mass terms predict a rather different KK spectrum for $d \sim 1$. A very large Majorana mass $d \gg 1$ does not shift the complete KK spectrum to higher values. The KK masses in this case depend in an oscillatory way on d . In the case of flat extra dimensions this behavior was already found in ref. [18].

One could ask under what conditions the bulk and boundary Majorana mass terms could be responsible for the observed small neutrino masses m_ν , once the bulk fermion field is identified with a SM neutrino. Because of the SM gauge invariance, the Majorana mass term must arise from an SU(2) triplet (either elementary or from two doublets). The gauge hierarchy problem requires Higgs fields and therefore the

Majorana mass term to be localized at the TeV-brane. We have studied this scenario in ref. [9], finding that a tuning of order 10^{-3} to 10^{-9} , depending on k/M_{Pl} , is needed to generate sub-eV neutrino masses. The neutrinos should be localized towards the Planck-brane.

Could the Majorana mass terms explain an eV-scale mass for a sterile neutrino? The bulk mass is certainly not a convincing possibility, since the small sterile neutrino mass has to be put in by hand in the 5D action. A natural value for the sterile neutrino mass could be expected to be comparable to the KK scale (TeV-size). If the Majorana mass is localized at the Planck-brane, small sterile neutrino masses can be achieved by localizing the fermion towards the TeV-brane. From eq. 16 we can read off that for $c \approx -1/2$ sub-eV masses are possible for $d \sim 1$. Since the neutrino is sterile, such a small value of c is not disfavored by electroweak observables [12]. If the Majorana mass is localized on the TeV-brane, small sterile neutrino masses can be produced by localizing the fermion towards the Planck-brane with $c \approx 1$.

In the next section we discuss how a Planck-brane Majorana mass assigned to a “right-handed” bulk neutrino leads to a satisfactory seesaw mechanism. Realistic neutrino masses can be accommodated without introducing any small numbers.

4 The seesaw mechanism in warped geometry

The seesaw mechanism provides a tiny mass for the SM neutrinos ν_L by coupling them to heavy right-handed neutrinos N [21]

$$M_\nu = \frac{\lambda_N^2 \langle H \rangle^2}{M_N}. \quad (22)$$

Here M_N denotes the Majorana mass for the right-handed neutrinos and $\lambda_N \nu_L N H$ is neutrino Yukawa interaction. Taking $M_\nu \sim 50 \text{ meV}$ (of the order of the atmospheric neutrino mass splitting $\sqrt{\Delta m_{\text{atm}}^2}$ [19]), one finds $M_N \sim \lambda_N^2 \cdot 6 \times 10^{14} \text{ GeV}$. For $0.01 \lesssim \lambda_N \lesssim 1$ this points to an intermediate scale for the right-handed Majorana mass.

Naively it seems problematic to implement the seesaw mechanism in a warped extra dimension. We have seen in the previous section that despite assigning a Planck-size ($d \sim 1$) Majorana mass to a bulk fermion, its lowest KK states have a mass of order $k\Omega^{-1}$, which is in the TeV region. However, the KK mass is (almost) Dirac-like. Inserting it into eq. (22) does not lead to the correct light neutrino mass.

In the following we study the coupling of two bulk fermion fields ν and N , corresponding to left- and right-handed neutrinos. The generalization to three generations is straightforward. Lepton number is broken by the Majorana mass of N , which we assume is localized at the Planck-brane, i.e. $m_M(N) = d \cdot \delta(y)$. Both fields may have bulk Dirac masses indicated by c_ν and c_N . Let us first discuss the situation along the lines of eq. (17), which means leaving out the Majorana mass (and the Yukawa interaction) in the KK reduction of N . From the KK reduction

of the left-handed neutrino field ν we obtain a left-handed zero mode $\nu_L^{(0)}$, corresponding to the SM neutrino, and an infinite tower of left- and right-handed KK excited states $\nu_L^{(m)}$ and $\nu_R^{(m)}$. The sterile (right-handed) neutrino decomposes into the right-handed zero mode $N_R^{(0)}$ and the KK excited states $N_L^{(m)}$ and $N_R^{(m)}$. In the basis of $(\nu_L^{(0)}, \bar{N}_R^{(0)}, \nu_L^{(1)}, \bar{\nu}_R^{(1)}, \bar{N}_R^{(1)}, N_L^{(1)}, \dots)$ the general mass matrix takes the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & C_{00} & 0 & 0 & C_{01} & 0 & \cdots \\ C_{00} & A_{00} & C_{10} & 0 & A_{01} & 0 & \cdots \\ 0 & C_{10} & 0 & D_{\nu,1} & C_{11} & 0 & \cdots \\ 0 & 0 & D_{\nu,1} & 0 & 0 & C_{o,11} & \cdots \\ C_{01} & A_{01} & C_{11} & 0 & A_{11} & D_{N,1} & \cdots \\ 0 & 0 & 0 & C_{o,11} & D_{N,1} & B_{11} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (23)$$

Here $D_{\nu,m}$ and $D_{N,m}$ denote the KK masses of ν and N , respectively. The mass terms A_{mn} and B_{mn} are defined as in eq. (18). Because we have taken a boundary Majorana mass, B_{mn} vanishes. The mass terms

$$\begin{aligned} C_{mn} &= \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_N^{(5)} H(y) f_{L,m}^{(\nu)}(y) f_{R,n}^{(N)}(y) \\ C_{o,mn} &= \int_{-\pi R}^{\pi R} \frac{dy}{2\pi R} \lambda_N^{(5)} H(y) f_{R,m}^{(\nu)}(y) f_{L,n}^{(N)}(y) \end{aligned} \quad (24)$$

arise from the Yukawa interaction with 5D coupling $\lambda_N^{(5)}$ after electroweak symmetry breaking. We take the Higgs profile to be strictly confined to the TeV-brane so that $C_{o,mn}$ vanish. Lepton number is violated only by the entries A_{mn} and B_{mn} .

We can compute the light neutrino mass by truncating the mass matrix (23). Taking more and more KK states into account, it can be checked numerically that the procedure indeed converges. In first approximation the light neutrino mass is found to be

$$m_\nu \approx \frac{C_{00}^2}{A_{00}} \left(1 - \frac{C_{00}^2 A_{11} + C_{01}^2 A_{00} - 2C_{00} C_{01} A_{01}}{D_{N,1}^2 A_{00}} + \dots \right). \quad (25)$$

The first term of this result is completely analogous to the ordinary seesaw formula (22). The seesaw scale turns out to be A_{00} , the mass of the heavy “chiral” mode in the spectrum of N , which was discussed in the previous section. The relevant Dirac mass in the numerator arises from the two zero modes. The KK masses of the excited states do not show up in the leading term since they are Dirac-like. They appear as corrections of order $\mathcal{O}(C^2/D^2)$ in eq. (25). The mass terms from the electroweak symmetry breaking C_{ij} are in the same range as the charged lepton masses, while the KK scale is TeV-size. We thus are left with tiny corrections to the

seesaw formula of order 10^{-6} . A related version of a warped seesaw mechanism was recently discussed in ref. [20], where the Higgs field was identified with a slepton in a (partly) supersymmetric setup.

The system can of course also be analyzed in the basis where the Majorana mass is included in the KK decomposition.⁴ The disadvantage of this procedure is that the states of the KK tower of N are no longer strictly Dirac-like and contribute to the light neutrino mass. Therefore one has to sum up all contributions up to the heavy “chiral” state in the spectrum. Depending on the size of the Majorana mass and the fermion locations (i.e. c parameters), the number of relevant states can be up to order Ω .

5 Discussion

The warped seesaw mechanism we just described generates sub-eV Majorana masses for the SM neutrinos. However, it is not their only source. In ref. [9] we discussed neutrino masses from the dimension-5 interaction $(1/M)HHLL$. Here we assume that this contribution is negligible due to a small coefficient multiplying the dimension-5 operator. One can also think of suppressing the dimension-5 operator by imposing lepton number symmetry, broken only at the Planck-brane. (This may occur, for instance, through spontaneous violation on the Planck-brane.)

The quantities C_{00} and A_{00} in the seesaw formula (25) depend on the fermion location. Moving the right-handed neutrino, i.e. its former zero mode, closer towards the TeV-brane, we can diminish A_{00} . At the same time C_{00} , which also depends on the location of ν , increases. This freedom allows us to generate a neutrino mass of the order of $\sqrt{\Delta m_{\text{atm}}^2}$, even with a Planck-size Majorana mass as input. In order to minimize deviations from electroweak observables, the SM fermions, and hence the neutrinos, should be localized towards the Planck-brane [12]. Taking therefore $c_\nu \gtrsim 1/2$, the right-handed neutrino should be localized at $0 \lesssim c_N \lesssim 1/2$ to generate the observed neutrino masses. In this range of parameters we have

$$\begin{aligned} A_{00} &= dk \left(\frac{1}{2} - c_N \right) \Omega^{2c_N-1} \\ C_{00} &= 2lv_0 \left(c_\nu - \frac{1}{2} \right)^{1/2} \left(\frac{1}{2} - c_N \right)^{1/2} \Omega^{-c_\nu-1/2} \end{aligned} \quad (26)$$

and the light neutrino mass (25) at leading order is given by

$$m_\nu \approx \frac{4l^2 v_0^2}{dk} \left(c_\nu - \frac{1}{2} \right) \Omega^{-2(c_\nu+c_N)} \quad (27)$$

where we have used $m_M = d \cdot \delta(y)$, $H(y) = v_0 \cdot \delta(y - \pi R)/\sqrt{k}$ and $\lambda_N^{(5)} = l/\sqrt{k}$.

⁴One could even include in the KK reduction the masses from electroweak symmetry breaking as well.

For definiteness we take $k = M_{\text{Pl}}$. To be consistent with electroweak constraints we assume $M_{KK} = 10 \text{ TeV}$ [12], which implies $kR = 10.83$. From the measured weak gauge boson masses we find $v_0 = 0.043k$ [6]. We take the SM neutrino location to be $c_\nu = 0.565$ [6]. We assume a light neutrino mass on the order of the atmospheric mass splitting $\sqrt{\Delta m_{\text{atm}}^2} = 50 \text{ meV}$. Then eq. (27) leads to a right-handed neutrino position of $c_N = 0.293$. We find an effective seesaw scale of $A_{00} = 3.9 \times 10^{11} \text{ GeV}$ and a Dirac mass of $C_{00} = 4.5 \text{ GeV}$. The right-handed neutrinos are sterile and can therefore be localized at $c < 1/2$ without disturbing the electroweak fit. From eq. (27) we also observe that m_ν only depends on $c_\nu + c_N$. The neutrino mass does not change if the left- and right-handed neutrinos are shifted in opposite directions by the same amount. The lowest lying KK states consist of an almost degenerate pair of sterile neutrinos with a mass of 8.5 TeV and a mass splitting of 0.1 MeV. The first KK excitations of the SM neutrinos have a mass of 10.3 TeV and are split by 1 MeV. The mass splittings may be affected by radiative corrections which we have neglected in our discussion. Clearly, the discussion can be extended to include the solar mass splitting.

In the mass matrix (23) the SM neutrinos mix with the left-handed KK states of the sterile neutrinos, where the mixing angles are on the order of $\theta_n \approx C_{0n}/D_{N,n}$. This mixing changes the effective weak charge of the light neutrinos. The effective number of neutrinos contributing to the width of the Z boson is reduced to $n_{\text{eff}} = 3 - \sum \sin^2 \theta_n$. A similar effect occurs if a small Dirac mass for the SM neutrinos is generated by coupling them to right-handed neutrinos in the bulk [8, 9]. Measurements of the Z width impose the constraint $\delta n \lesssim 0.005$ [22]. For the parameter values discussed above we find $\delta n = 2 \cdot 10^{-6}$, well below the experimental sensitivity, but still much larger than in the ordinary 4D seesaw. The mixing is similar to the value we obtained for the model of ref. [9]. The admixture of sterile states becomes larger if the SM neutrinos are localized closer towards the TeV-brane, or if the KK scale is reduced.⁵

Similar to our discussion in ref. [9] the mixing between SM neutrinos and KK sterile neutrinos considerably enhances lepton flavor violating processes [23], such as $\mu \rightarrow e\gamma$. In the warped seesaw we expect the rates for such processes to be of the same order as in the model of Dirac neutrino masses, which were found to be several orders of magnitude below the experimental bound [9]. The branching ratio might be brought to an experimentally interesting range if the admixture of sterile states can be enhanced. Of course, the setup we discussed here is crucially different from the model of ref. [9] since the light neutrino mass is Majorana-like. Depending on the absolute value of the neutrino mass, this can be tested in neutrinoless double beta decay experiments [24].

Finally, we briefly discuss what happens if the Majorana mass for the singlet neutrino is introduced away from the Planck-brane. If a Majorana mass of order M_{Pl} is localized at the TeV-brane, it will be warped down to TeV-size. We expect

⁵For possibilities to lower the KK scale see refs. [15, 16].

the “light” neutrino mass then to be of order $\text{GeV}^2/\text{TeV} \sim \text{MeV}$. The situation is similar if the Majorana mass is placed in the bulk. Taking it to be of order M_{Pl} , it completely destroys the vector-like nature of the KK excitations, which emerge as Majorana particles with TeV-scale masses. Again we end up with neutrino masses in the MeV-range. Thus the warped seesaw prefers the Majorana mass to be localized at the Planck-brane.

6 Conclusions

We have studied the seesaw mechanism in a warped geometry framework. Sterile (“right-handed”) neutrinos are introduced in the bulk which couple to the SM neutrinos. Lepton number is broken by a Planck-size Majorana mass for the sterile neutrinos. If the Majorana mass is confined to the Planck-brane, a heavy mass scale for the seesaw is generated. The effective seesaw scale is of order $M_P \exp((2c - 1)\pi k R)$ and depends on the location, i.e. 5D Dirac mass parameter c , of the sterile neutrino in the bulk. For $c < 1/2$ intermediate values of the seesaw scale emerge. For $c \approx 0.3$ light neutrinos masses needed to explain the atmospheric and solar neutrino oscillations are obtained without introducing any small parameters. The KK spectrum consists of the almost degenerate excitations of the SM and sterile neutrinos, which have masses in the TeV-range. It remains to be seen if the appearance of an intermediate mass scale for right handed neutrinos allows one to implement a successful mechanism of leptogenesis to account for the baryon asymmetry of the universe.

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